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Short range linear potential in 3D lattice compact QED

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Abstract

We study the static potential between electric charges in the finite temperature three-dimensional compact gauge theory on the lattice. We show that in the deconfinement phase at small separations between the charges the potential contains a linearly rising piece which goes over into the Coulomb potential as the distance between the charges is increased. The linear potential is due to the gas of magnetic dipoles which are realized as monopole–anti-monopole bound states. © 2001 Published by Elsevier Science B.V.

1. Introduction

The compact Abelian gauge theory in three space-time dimensions at finite temperature has two phases separated by the phase transition [2–4]. At low temperature the electric charges are confined, while at high temperature the confinement disappears. These properties of the model originate from the monopole dynamics, the monopoles being topological defects appearing due to the compactness of the gauge group.

In the low temperature phase the Abelian monopoles form a plasma of magnetic charges. Due to the long-range nature of the gauge fields associated with the monopoles the behavior of the plasma is Coulombic. In the weak coupling regime the plasma is sufficiently dilute to warrant the use of mean field methods in order to analyze the non-perturbative behavior of the system. As it was shown in Ref. [5] test particles of op-

posite electric charges being immersed in the magnetic Coulomb plasma experience confining forces at large separations. The confinement appears because of the formation of a string-like object between the charges. The string has a finite thickness of the order of the inverse Debye mass and a finite energy per unit of string length (“string tension”).

At high temperature the physics of monopoles changes: monopoles and anti-monopoles form magnetically neutral bound states [6]. These states are filling up the vacuum with a neutral dipole plasma and the confinement of electric charges disappears. This phenomenon can be explained in two ways. First, the field of the magnetic dipoles is much weaker at large distances compared to the field of the monopoles and thus the dipole is unable to induce a non-zero string tension. The second explanation is that in the dilute dipole plasma the Debye screening is absent [7] and effectively this corresponds to an infinitely thick confining string. Since the electric flux of the string is a conserved quantity, it is no more concentrated in a

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small region (“core of the string”) and thus the string “dissolves” as we go from low to high temperatures.¹

Despite the fact that the dipoles themselves are unable to create the confining string, they may still be responsible for the non-perturbative physics of electric charges. The string tension corresponding to the linear part of the long distance potential of electric charges in a pure magnetic monopole gas increases with addition of the magnetic dipoles to the monopole gas [9]. At short distances the potential of electric charges in a pure magnetic dipole gas contains a piece which rises linearly with the distance between the charges [10].

Note that non-trivialities in a short distance potential appear also in the zero-temperature theory. According to Ref. [11], at distances R much smaller than the correlation length of the monopole–anti-monopole plasma the potential contains a perturbative contribution due to one-photon exchange plus a non-perturbative piece proportional to R^α , $\alpha \approx 0.6$. A non-trivial short-distance potential may have many physically interesting consequences, see, e.g., Ref. [12] for a discussion in the context of QCD and other theories. The monopole binding in 3D compact QED is qualitatively similar to the formation of the instanton molecules in the high temperature phase of QCD suggested to be responsible for the chiral phase transition [13].

The structure of this Letter is as follows. In the next section we review some results concerning the physics of the dipole gas in the continuum following Ref. [10]. Section 3 contains the results of our simulation of the compact Abelian gauge model in three dimensions. We show that indeed the short-distance potential contains a linear piece, which can be explained as due to the non-trivial dynamics of the gas of lattice dipoles. Our conclusions are summarized in the last section.

2. Dipole gas in continuum

A magnetic dipole is a magnetically neutral localized pair of a monopole and an anti-monopole separated by a distance r . The magnetic moment of the dipole is $\vec{\mu} = g_m \vec{r}$ where g_m is the magnetic charge of the constituent monopole and \vec{r} is the relative position

vector pointing from the anti-monopole to the monopole. If the typical distance between dipoles is much larger than the dipole size r , then the dipoles may be treated as point-like particles. This condition can be written as follows:

$$\xi = \rho^{1/3} r \ll 1, \quad (1)$$

where ρ is the mean density of the dipole gas. We shall see below that in our calculations this condition is always fulfilled.

The action of two interacting point-like dipoles with magnetic moments $\vec{\mu}_a$ and $\vec{\mu}_b$ located at positions \vec{x}_a and \vec{x}_b , respectively, is given by the formula:

$$V(\vec{\mu}_a, \vec{\mu}_b; \vec{x}_a, \vec{x}_b) = (\vec{\mu}_a \cdot \vec{\partial})(\vec{\mu}_b \cdot \vec{\partial}) D_{(3D)}(\vec{x}_a - \vec{x}_b), \quad (2)$$

where $D_{(3D)}(\vec{x}) = 1/(4\pi|\vec{x}|)$ is the propagator for a scalar massless particle in three dimensions.

Below we consider the case of the fixed absolute values of the dipole moments, $\mu_a = \mu = \text{const}$. The case of fluctuating dipole moments is considered in Ref. [10].

The statistical sum of the dilute dipole gas can be written as follows:

$$\mathcal{Z} = \sum_{N=0}^{\infty} \frac{\zeta^N}{N!} \int d^3x_1 \int d^3\mu_1 \cdots \int d^3x_N \int d^3\mu_N \times \exp \left[-\frac{1}{2} \sum_{\substack{a,b=1 \\ a \neq b}}^N V(\vec{\mu}_a, \vec{\mu}_b; \vec{x}_a, \vec{x}_b) \right], \quad (3)$$

where ζ is the dipole fugacity. As in the case of the monopole gas [5] the dipole fugacity is a non-perturbative quantity, since $\zeta \sim e^{-S_0}$, where $S_0 \sim g_e^{-2}$ is the action of a single dipole and $g_e = 2\pi/g_m$ is the fundamental electric charge in the theory.

The partition function (3) can be rewritten as follows [10],

$$\mathcal{Z} = \int D\chi \exp \left\{ - \int d^3x \left[\frac{1}{2} (\vec{\partial}\chi)^2 - 4\pi\zeta \frac{\sin(\mu|\vec{\partial}\chi|)}{\mu|\vec{\partial}\chi|} \right] \right\}, \quad (4)$$

where $|\vec{\partial}\chi| = \sqrt{\vec{\partial}\chi^2}$. Rescaling the field, $\chi \rightarrow (4\pi\zeta)^{-1/3} \mu^{-1} \chi$, and the coordinates, $x \rightarrow (4\pi\zeta)^{-1/3} x$,

¹ Another mechanism of the phase transition in the Georgi–Glashow model due to magnetic vortex dynamics is discussed in Ref. [8].

we immediately realize that the dynamics of the gas is controlled by the dimensionless constant

$$\lambda = 4\pi\zeta\mu^2. \quad (5)$$

The vacuum expectation value of the dipole density, $\rho_d(x) = \sum_i \delta^{(3)}(x - x_i)$, is

$$\rho_d \equiv \langle \rho_d(x) \rangle = 4\pi\zeta(1 + O(\lambda)), \quad (6)$$

where the last equality is valid for small couplings λ . In this regime the coupling λ is proportional to the density of the dipoles, $\lambda = \rho_d\mu^2$, up to $O(\lambda^2)$ terms. Therefore the smallness of the parameter λ can be interpreted as a requirement for the density of the dipole moments to be small:

$$\rho_d\mu^2 \ll 1. \quad (7)$$

The static interaction of particles with electric charges g_e is the sum of the perturbative contribution from the one-photon exchange and the contribution from the dipole gas, respectively:

$$V(R) = \frac{g_e^2}{2} D_{(2D)}(R) + E^{\text{gas}}(R),$$

$$E^{\text{gas}}(R) = - \lim_{T \rightarrow \infty} \frac{1}{T} \langle W(C_{R \times T}) \rangle, \quad (8)$$

where $C_{R \times T}$ stands for the rectangular $R \times T$ trajectory of the test particle and $D_{(2D)}(R) = -(2\pi)^{-1} \times \log(mR)$ is the two-dimensional propagator for a scalar massless particle, m is a regulator of the dimension of mass. We consider here test particles with an electric charge equal to one unit of the fundamental charge. The case of particles of arbitrary charges is discussed in Ref. [10].

At small distances between the test electric charges, $R \ll r$, the behavior of the potential was shown to be as follows [10]:

$$E_{\text{cl}}^{\text{gas}}(R) = \sigma R \left[1 + O\left(\frac{R}{r}\right) \right],$$

$$\sigma = \pi^3 \zeta r = \frac{\pi^2}{4} \rho_d r, \quad R \ll r, \quad (9)$$

where σ is given to leading order in λ .

At large distances, $R \gg r$, the classical energy of the electric charges in the dipole gas grows logarithmically:

$$E_{\text{cl}}^{\text{gas}}(R) = \frac{8\pi^2}{3} \zeta r^2 \log\left(\frac{R}{r}\right) \left[1 + O\left(\frac{R}{r}\right) \right], \quad R \gg r. \quad (10)$$

According to Eq. (8) the dipole gas non-perturbatively renormalizes the coupling constant g_e at large distances and the full potential (8) has the following behavior:

$$V(R) = \frac{g_e^2}{2\epsilon} D_{(2D)}(R),$$

$$\epsilon = 1 + \frac{1}{3}\lambda + O(\lambda^2), \quad R \gg r, \quad (11)$$

where ϵ is the dielectric constant (permittivity).

The linear term at small distances (9) is a result of the interaction of the dipole clouds which surround the external electric sources. Indeed, in three dimensions the static test charge trajectories j_C may be considered as electric currents running along the “wires” j_C . These currents induce a magnetic field which encircles the trajectories and is defined to leading order by the classical Maxwell equations. The closer to the current the larger the magnetic field is. Since the energy of a magnetic dipole becomes lower with increasing magnetic field strength, the density of the magnetic dipoles should increase towards the position of the electric test charges. Therefore the dipoles form clouds near test particles and the interaction of these clouds is responsible for the non-perturbative part of the inter-particle potential.

3. Compact QED in 3D: numerical results

We consider the 3D compact $U(1)$ gauge model with the standard Wilson action,

$$S = -\beta \sum_P \cos \theta_P,$$

$$\theta_{P_{x,\mu\nu}} = \theta_{x,\mu} + \theta_{x+\hat{\mu},\nu} - \theta_{x,\nu} - \theta_{x+\hat{\nu},\mu}, \quad (12)$$

where θ_P is the Abelian field strength tension defined on the plaquette P . The tensor is constructed from the Abelian compact fields θ_l attached to the links. The lattice coupling is related to the electric charge g_e in the following way,

$$\beta = \frac{1}{ag_e^2}, \quad (13)$$

a is the lattice spacing. Our numerical results are obtained on the lattice of size $32^2 \times 8$. The finite temperature phase transition corresponds to the lattice

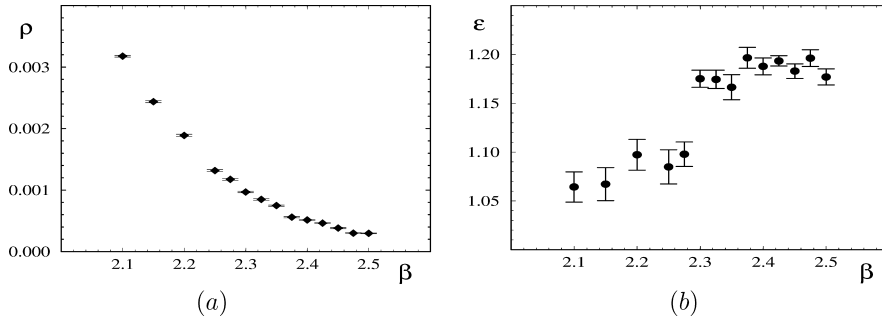


Fig. 1. (a) the density of the Abelian monopoles, ρ , vs. lattice coupling β ; (b) vacuum dielectric constant (permittivity) ϵ .

coupling $\beta_c = 2.3$, Ref. [4]. The high-temperature phase corresponds to $\beta > \beta_c$.

We do realize that our results cannot be expected to work in the confinement phase. Therefore, when fits are made only the numerical results for $\beta > \beta_c$ are used. Still, we consider it worthwhile to show the numerical results in a larger domain of β values as they illustrate the qualitative difference between the two regimes.

We define the Abelian monopole density ρ in the standard way [1]:

$$\rho_x = \frac{1}{2\pi} \sum_{P \in c_x} (-1)^P [\theta_P]_{2\pi}. \quad (14)$$

The position of the monopole current x belongs to the dual lattice. The summation is taken over the boundary of the three-dimensional cube c_x which has the point x as its center. The left hand side of Eq. (14) is the divergence of the magnetic flux. Since the theory is compact, the physical field strength tensor in Eq. (14) is defined modulo 2π .

We show the behavior of the density ρ of the Abelian monopoles vs. β in Fig. 1(a). The density ρ decreases rapidly with increasing of β (i.e., with increasing of the temperature). Note that in the deconfinement phase, $\beta > \beta_c$ the density of the monopoles is non-zero. Analysis of the gauge field configurations gives that the monopoles appear as tight monopole–anti-monopole bound states in which the constituents are separated basically by one or two lattice spacings a .

We study the potential between oppositely electrically charged particles measuring the correlations of

two Polyakov lines, $L(x)$, separated by a distance R ,

$$V(R) = -\log \langle L(0)L^+(R) \rangle. \quad (15)$$

The lattice potential $V(R)$ is fitted by the following formula:²

$$V(R) = -\frac{1}{\beta_{\text{fit}}} D(R) - \log \{ \cosh[\sigma(R - L/2)] \} + C, \quad (16)$$

where the effective coupling, β_{fit} , the short string tension, σ , and the additive energy renormalization, C , are the fitting parameters. The function $D(R) \equiv D(R, 0)$ is the two-dimensional lattice propagator for a scalar massless particle:

$$D(\vec{x}) = \frac{1}{2L^2} \times \sum_{\substack{k_1, k_2=0 \\ \vec{k}^2 \neq 0}}^{L-1} \frac{e^{2\pi i \vec{k} \cdot \vec{x}/L}}{2 - \cos(2\pi k_1/L) - \cos(2\pi k_2/L)}, \quad (17)$$

where the zero mode, $k_1 = k_2 = 0$, is excluded. We are using the massless propagator since in the high temperature phase (which we are interested in) the potential between electric charges is long ranged according to Eq. (11) (the dipoles are unable to induce a finite correlation length). Function (16) fits the numerically obtained potential (15) with $\bar{\chi}^2 = \chi^2/\text{d.o.f.} \sim 1$ up to distances $ra \sim 8$, while at large distances the function $\bar{\chi}^2$ become unacceptably large.

² Note that the potential defined in this way may contain contributions from excited states in addition to the ground state potential.

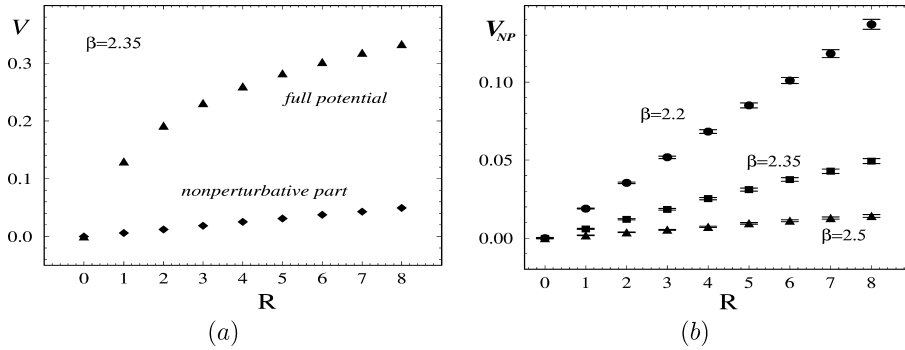


Fig. 2. (a) nonperturbative part of the test particle potential, Eq. (19), vs. full potential, for $\beta = 2.35$, and (b) the non-perturbative parts of the potential for various values of β .

Thus we are fitting the potential (15) by the function (16) at the distances $R = 0, \dots, 8$.

The second term in Eq. (16) corresponds to the linear term modified by finite volume corrections. In the limit $L \rightarrow \infty$ this term transforms to the ordinary linear term σR . Note that the finite volume corrections in the first term are already taken into account since according to Eq. (17), $D(R + L) = D(R)$.

The coupling β_{fit} used as a fitting parameter in Eq. (16) should be equal to the “bare” coupling, β , in first order perturbation theory. However, due to the non-perturbative corrections coming from the dipole gas [10], this coupling gets renormalized. The ratio of the renormalized and bare couplings, according to Eqs. (11), (13) is the dielectric constant (permittivity):

$$\epsilon(\beta) = \frac{\beta_{\text{fit}}(\beta)}{\beta} = \frac{g_{\text{e,bare}}^2}{g_{\text{e}}^2}, \quad (18)$$

which is plotted in Fig. 1(b). In the deconfinement region, $\beta > \beta_c$, close to the phase transition the dielectric constant ϵ is approximately equal to 1.2. A similar investigation of the charge renormalization in four-dimensional zero-temperature QED has been done in Ref. [14].

The non-perturbative part of the inter-particle potential is given by the full potential minus the perturbative one-photon exchange,

$$V_{\text{NP}}(R, \beta) = V_{\text{exp}}(R, \beta) + \frac{1}{\beta_{\text{fit}}(\beta)} D(R). \quad (19)$$

In Eq. (8) the non-perturbative part of the potential is solely due to the dipole gas contribution, $E^{\text{gas}}(R)$,

and, according to our fits it is linear. We show the full potential and the non-perturbative part of the potential $\beta = 2.35$ in Fig. 2(a). Indeed, we observe that the non-perturbative part is linear and is substantially smaller than the perturbative one. The non-perturbative parts of the potential for various β are shown in Fig. 2(b).

The tension of the short string σ , defined in Eq. (16) is plotted vs. β in Fig. 3(a). Note that σ is a decreasing function of β (or, equivalently, of temperature). The short distance string tension at the high temperature side is non-zero due to the magnetic dipoles dynamics discussed in Section 2, Eq. (9).

In order to check the consistency of the results obtained, with the dipole gas picture we first estimate the dipole size from the string tension and the monopole density using Eq. (9). Taking into account the fact that the density of the dipoles, ρ_d , is half the density of the constituent monopoles, ρ , we get:

$$r = \frac{8\sigma}{\pi^2 \rho}. \quad (20)$$

We evaluate r by this formula and plot it as a function of β in Fig. 3(b). In the deconfinement phase,³ $\beta > \beta_c$, the dipole sizes become smaller as β increases, in qualitative agreement with the theoretical estimates. Note that according to Eq. (20) at large β the dipole sizes are of the order of 4 or 5 lattice

³ In the confinement phase formula (20) should not work since in this phase a fraction of the monopoles is not bounded in dipole states.

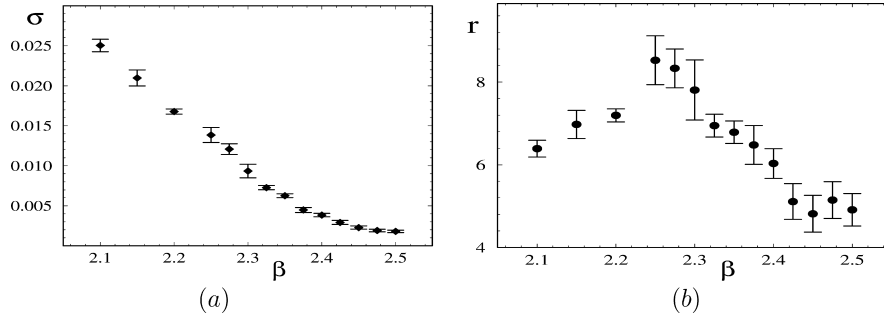


Fig. 3. (a) the average distance between constituent monopoles in a dipole state, Eq. (20); (b) the short distance string tension obtained with the help of the fit (16).

spacings while the observed sizes are of the order of $1, \dots, 2$ spacings.

There are two reasons which may explain observed quantitative difference in r of a factor of 3. The first reason might be that the analytical formulae for the dipole density (6) and the string tension (9) are not applicable. This might be caused by a violation of the dipole diluteness condition (1) or of the requirement for the density of the dipole moments to be small, Eq. (7).

Second, the distribution of the dipole magnetic moment for a real dipole state is unknown while in our theoretical estimations we have assumed fixed dipole moments for definiteness. The knowledge of the distribution is essential in region near the phase transition in which the dipole gas is not dilute. However since the strength of the monopole–anti-monopole potential is a linear function of the temperature [6] at large temperatures then the size of the dipole pair should decrease with increasing of the temperature. The density of the monopole pairs is a decreasing function of the temperature, Fig. 1(a). Therefore as the temperature increases the number and the size of the dipoles becomes smaller and the applicability of theoretical formulae gets better. Below we discuss the applicability quantitatively.

The quantity characterizing the diluteness of the gas, ξ , Eq. (1), is of the order of 0.1 for the measured distances between monopole constituents. However, the dipole sizes found with the help of Eq. (20) give $\xi \sim 0.3$. The last relation indicates that the point-like dipole gas approach might be “on the edge of applicability”. However, the density of the dipoles as well as the dipole size decrease with the temperature

and therefore the condition (1) should be satisfied better as temperatures increases.

The other assumption which has been used in the derivation of the analytic expressions for the short string tension σ , Eq. (9), and the permittivity ϵ , Eq. (11), is the smallness of the parameter λ , Eq. (5). Using Eqs. (6), (9), (13) we can express λ as a function of the monopole density, the string tension and the lattice coupling:

$$\lambda = \frac{32\sigma^2\beta}{\pi^2\rho}. \quad (21)$$

With the help of this equation we get that λ is a decreasing function of β which is equal to 0.8 at $\beta = \beta_c$ and decreases to 0.3 at $\beta = 2.5$. Thus near the phase transition and even at the largest studied values of β the requirement (7) fails to be fulfilled. However, at large β the coupling λ is about to be small enough for the theoretical formulae for the short string tension σ , Eq. (9), and the permittivity ϵ , Eq. (11), to start working.

Thus we may conclude that our theoretical formulae may work qualitatively rather than quantitatively. As we have seen above this is indeed the case: the linear potential is observed while the mismatch in the self-consistency check for the dipole sizes for the largest value of β is a factor of 3. Better agreement between theoretical and numerical values can be observed for the dielectric permittivity. Indeed, according to Eq. (11), $\epsilon_{th} - 1 = \lambda/3 \approx 0.118$ while numerical data shown in Fig. 1(b) gives the result $\epsilon_{num} - 1 = 0.177$.

4. Conclusions

We have observed the non-perturbative piece in the inter-particle potential in high temperature compact QED in three dimensions to rise linearly with the distance. This effect is in qualitative agreement with the predictions of the point-like magnetic dipole gas model with fixed dipole sizes. The electric permittivity of the vacuum was also calculated and turns out to be very close to the theoretical value.

These results may have interesting applications for the physics of the gauge theories possessing monopole topological excitations which form bound states. One of these theories is the electroweak model in which the formation of the Nambu monopole–anti-monopole pairs has been observed in the high temperature phase [15].

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